Mark Scheme

Summer 2023

Pearson Edexcel GCE
In A Level Further Mathematics (9FM0)
Paper 3D Pure Mathematics

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Summer 2023
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

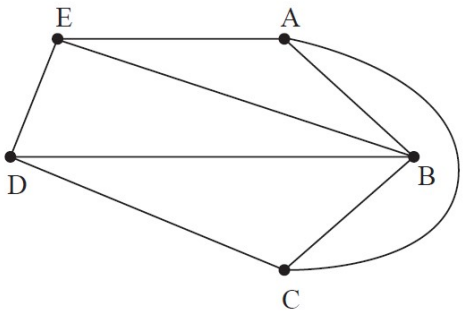
- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>1.(a)</td><td>Graph G is neither as there are more than two vertices of odd degree</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>e.g. A – B – C – D – E – A</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(c)</td><td>G <u>is</u> planar as it can be drawn with no arcs intersecting/crossing each other
e.g.
 
</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(d)</td><td>2</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(e)</td><td><table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>10</td><td>15</td><td>∞</td><td>5</td></tr>
<tr><td>B</td><td>10</td><td>-</td><td>3</td><td>8</td><td>4</td></tr>
<tr><td>C</td><td>15</td><td>3</td><td>-</td><td>2</td><td>∞</td></tr>
<tr><td>D</td><td>∞</td><td>8</td><td>2</td><td>-</td><td>7</td></tr>
<tr><td>E</td><td>5</td><td>4</td><td>∞</td><td>7</td><td>-</td></tr>
</table></td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(f)</td><td><table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th><th>E</th></tr>
<tr><td>A</td><td>-</td><td>[9]</td><td>[12]</td><td>[12]</td><td>5</td></tr>
<tr><td>B</td><td>[9]</td><td>-</td><td>3</td><td>5</td><td>4</td></tr>
<tr><td>C</td><td>[12]</td><td>3</td><td>-</td><td>2</td><td>7</td></tr>
<tr><td>D</td><td>[12]</td><td>5</td><td>2</td><td>-</td><td>7</td></tr>
<tr><td>E</td><td>5</td><td>4</td><td>7</td><td>7</td><td>-</td></tr>
</table></td><td>M1
A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(7 marks)</td></tr>
</table>

Notes for Question 1

(a)
B1: 'Neither' together with a correct reason (ignore irrelevant statements but do not isw incorrect statements)

Examples of B1:

- G contains more than 2 odd nodes
- G contains 4 odd nodes (**or** stating that A, C, D and E are odd)
- G contains (only) 1 even node
- The number of odd degree nodes in G is not 0 or 2

Examples of B0

- G contains odd nodes
- G contains at least 2 odd nodes
- G contains 4 nodes of degree 3 (not linking this to 'odd')

(b)
B1: CAO - must begin and end at the same node and include every other node in the graph exactly once. Accept if given in terms of arcs e.g. AC, CB, BD, DE, EA

(c)
B1: Correct answer of planar + justification

Examples of correct justification:

- AC can be moved outside **or** EB, DB can be moved outside
- AC(O), BE(I), BD(I) (or vice-versa) accept just AC(O) **or** BE(O) and BD(O)
- A correct drawing of the graph as planar (condone nodes not being labelled)

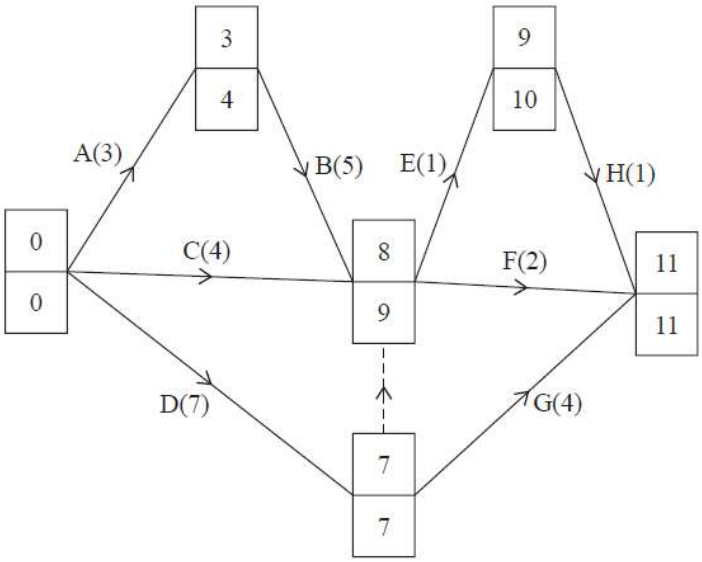
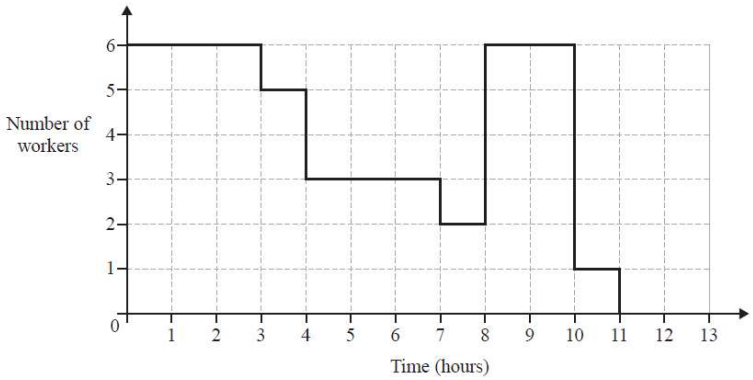
Examples of insufficient justification:

- Move arc BE (or BD) outside (need to mention both)
- Move arcs outside so that no arcs cross each other (must give specific examples of which arc(s) are being moved outside)
- Comments about moving nodes

(d)
B1: CAO (2 only)

(e)
B1: CAO - no blanks entries (apart from the lead diagonal) and must include ∞ in cells AD, CE, DA, EC)

(f)
M1: No change in the fifth row and fifth column with at least **two** values reduced correctly (no blank entries – apart from the lead diagonal)
A1: CAO for the final iteration

Question	Scheme	Marks	AOs
2. (a)	(i)  	M1 A1	1.1b 1.1b
	(ii) 11 (hours)	A1ft	2.2a
		(3)	
(b)	 (i)	M1 A1 A1	1.1b 1.1b 1.1b
	(ii) Six workers are required in the time intervals 0 – 3 and 8 – 10	A1	2.2a
		(4)	
			(7 marks)

Notes for Question 2

(a)(i)

M1: All top boxes and all bottom boxes completed. Values generally increasing left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0s at the source node and/or their 11 in the bottom box at the sink node for M only. Condone one rogue value in top boxes and one rogue value in bottom boxes. For a rogue in the top boxes if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value (with a similar definition for bottom boxes but in reverse)

A1: CAO - Top boxes and Bottom boxes (**all** completed)

(ii)

A1ft: Follow through their final early event time (the final late event time does not need to be the same as the final early event time to award this mark) – units not required and ignore incorrect units (just mark the value stated)

(b)(i) Do not consider the placement of the activities (if shown) in the histogram – consider only the placement of each vertical bar

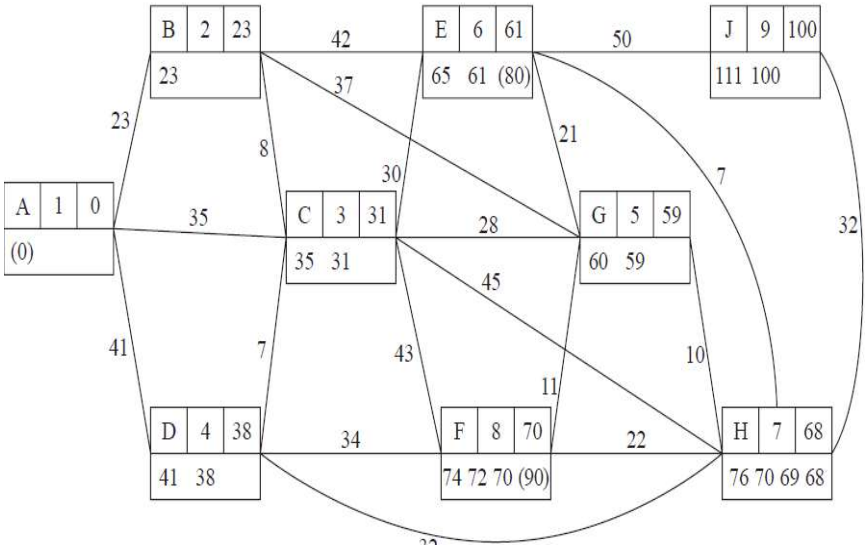
M1: Plausible histogram (correct up to time 4) with no holes or overhangs (must go to at least 10 on the time axis)

A1: Histogram correct to time 8

A1: Histogram correct from time 8 to time 11 with no activities taking place after time 11

(b)(ii)

A1: CAO (0 to 3 and 8 to 10) – allow any indication of an interval from 0 to 3 **and** 8 to 10 (so accept any use of $<, \leqslant$ or just a dash but not, for example, 0 to 2.999…)

Question	Scheme	Marks	AOs
3(a)	(i)  	M1 A1 (ABCD) A1 (GEH) A1ft (FJ)	1.1b 1.1b 1.1b 1.1b
	Shortest path from A to J is ABCEHJ	A1	2.2a
	Length of shortest path is 100	A1ft	2.2a
		(6)	
(b)	$0.0312\times\left(\frac{9000}{9}\right)^2$	M1	1.1a
	= 31 200 seconds therefore 520 minutes	A1	2.2a
		(2)	
			(8 marks)

Notes for Question 3

In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at D the working values must be 41 38 in that order (so 38 41 is incorrect)
It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(a) M1: A larger value replaced by a smaller value in the working values at **two** of C or D or E or F or G or H or J

A1: All values in A, B, C and D correct. Condone lack of 0 in A's working value

A1: All values G, E and H correct and the working values in the correct order. Penalise order of labelling only once per question (G, E and H must be labelled in that order and G must be labelled after A, B, C and D). Note that an additional working value of 80 at E after the 61 is not an error so 65 61 80 is fine, however, any other number or e.g. 80 65 61 in this order is incorrect and scores A0 in this part

A1ft: All values in F and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question.

To follow through F check that the working values at F follow from the candidate's final values for the nodes that are directly attached to F (which are C, D, G and H). For example, **if** correct then the order of labelling of nodes C, D, G and H are 3, 4, 5 and 7 respectively so the working values at F should come from C, D, G and H in that order. The first working value at F should be their 31 (the Final value at C) + 43 (the weight of the arc CF), the second working value at F should be their 38 (the Final value at D) + 34 (the weight of the arc DF), the third working value at F should be their 59 (the Final value at G) + 11 (the weight of the arc GF), the fourth working value need not be there (as 68 + 22 > 70) but do not penalise if seen. Repeat the process for J (which will have working values from E and H with the order of these nodes determined by the candidate's order of labelling at E and H)

A1: CAO - correct path from A to J (ABCEHJ) and not the path from J to A

A1ft: ft their final value at J only (if 100 stated and 100 is not the final value at J then **A0**)

(b)

M1: Complete, correct method – allow reciprocal e.g. $0.0312\times\left(\frac{9}{9000}\right)^2$ – allow slips in values only e.g. 0.312 for 0.0312. If using anything other than squared e.g. n^3 then **M0**. The correct answer in seconds, minutes or hours (8 hours 40 minutes) implies this mark (and if in minutes implies the A mark too)

A1: CAO – the exact value of 520 must be stated at some point (as question specifically asked for the answer in minutes) – condone lack of units (but if present must be correct)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>4. (a)</td><td>$132+x>120$ for all positive values of x therefore 3 bins is not possible</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td>(1)</td><td></td></tr>
<tr><td>(b)</td><td>$23<x\leqslant 28$</td><td>B1
B1</td><td>2.2a
2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td>(c)</td><td>Bin 1: 15 22 3
Bin 2: 9 23 5
Bin 3: x 4
Bin 4: 18 20
Bin 5: 13</td><td>M1
A1
A1</td><td>1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(3)</td><td></td></tr>
<tr><td>(d)</td><td>
<table>
<tr><td>15</td><td>22</td><td>3</td><td>9</td><td>23</td><td>x</td><td>5</td><td>4</td><td>18</td><td>20</td><td>13</td></tr>
<tr><td>x</td><td>15</td><td>22</td><td>3</td><td>9</td><td>23</td><td>5</td><td>4</td><td>18</td><td>20</td><td>13</td></tr>
<tr><td>x</td><td>15</td><td>22</td><td>9</td><td>23</td><td>18</td><td>20</td><td>13</td><td>5</td><td>3</td><td>4</td></tr>
<tr><td>x</td><td>23</td><td>15</td><td>22</td><td>9</td><td>18</td><td>20</td><td>13</td><td>5</td><td>4</td><td>3</td></tr>
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</table>
</td><td>M1
A1
A1ft
A1</td><td>1.1b
1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td>(e)</td><td>e.g. attempt at first-fit decreasing (or clearly considering the cases in the correct order)
Bin 1: x
Bin 2: 23 15 If 15 cannot go in Bin 1 then $x > 25$
Bin 3: 22 18
Bin 4: 20 13 If 13 cannot go in Bin 1 then $x > 27$</td><td>B1</td><td>3.1a</td></tr>
<tr><td></td><td>Therefore, x must be 28</td><td>dB1</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(2)</td><td></td></tr>
<tr><td colspan="4">(12 marks)</td></tr>
</table>

Notes for Question 4

(a)

B1: Correct reasoning for why 3 bins is not possible or at least 4 bins are required

Examples of B1:

- If $x > 23$ then $132 + x$ is at least 156 and $\frac{156}{40} = 3.9$ so 3 is not possible
- The lower bound for the number of bins is $\frac{156}{40} = 3.9$ so 3 is not possible
- If $x = 24$ then $\frac{156}{40} = 3.9$ so 3 is not possible (or comparing 156 with 120)
- If $x > 23$ then $132 + x$ is > 155 so you would need to have more than $\frac{155}{40} = 3.875$ bins so is not possible (condone 'at least' rather than 'more than' on the bod so condone comparing 155 with 120)
- 4 values are at least half the capacity of a bin (or at least 20) and they are 20, 22, 23 and x, so 3 so not possible (**must** explicitly state the four values but condone those candidates who imply that the 20, 22, 23 and the x are all greater than half the capacity of a bin)
- Comparing 132 with 120 and saying that 3 is not possible (no mention of x is required)
- x would have to be –12 for it to be possible (but not just that x would have to be negative)

(b)

B1: One correct inequality ($x > 23, x \geqslant 24, x \leqslant 28$ or $x < 29$)

B1: CAO ($23 < x \leqslant 28$ or $24 \leqslant x \leqslant 28$ or 29 with a strict inequality)

(c) NO MISREADS MARK EXACTLY TO THE SCHEME – if x is given a value then M1 only

M1: First five values placed correctly (underlined values) and the x not in Bin 1 or 2 – at least eight values placed (condone cumulative values for **M1** only)

A1: First eight values places correctly (underlined and boxed values with no repeated values)

A1: CSO no additional/repeated values

(d)

M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is **M0**). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then max of M1A1 only** – Bubble sort is not a MR and scores **M0**

A1: First two passes correct (pivots for third pass need not be chosen)

A1ft: Third and fourth passes correct (follow through from their second pass and choice of consistent pivots). They do not need to be choosing a pivot for the fifth pass for this mark. After their second pass their list must contain either 10, 11 or 12 numbers (so allow one additional or one missing number or a slip in one number e.g. the 5 becoming a 6)

A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) – both middle left and middle right require six passes

SC: If the list is sorted in ascending order, then award a maximum of M1A1A0A0 (so 2 marks) as in the scheme above even if the list is re-ordered after the sort is complete

Ascending (below is middle-right for middle-left the pivot is the 23):

15	22	3	9	23	***x***	5	4	18	20	13
15	22	3	9	23	**5**	4	18	20	13	$\underline{x}$
3	4	5	15	22	9	23	18	20	13	x

Middle left:

15	22	3	9	23	***x***	5	4	18	20	13
$\underline{x}$	15	22	3	9	**23**	5	4	18	20	13
$\underline{x}$	$\underline{23}$	15	22	3	9	**5**	4	18	20	13
$\underline{x}$	$\underline{23}$	15	22	**9**	18	20	13	$\underline{5}$	**3**	4
$\underline{x}$	$\underline{23}$	15	22	**18**	20	13	$\underline{9}$	$\underline{5}$	4	$\underline{3}$
$\underline{x}$	$\underline{23}$	**22**	20	$\underline{18}$	**15**	13	$\underline{9}$	$\underline{5}$	4	$\underline{3}$
$\underline{x}$	$\underline{23}$	$\underline{22}$	20	$\underline{18}$	$\underline{15}$	13	$\underline{9}$	$\underline{5}$	4	$\underline{3}$

(e)

B1: any clear indication that as the 15 does not fit in Bin 1 then $x + 15 > 40$ (condone for this mark just stating $x > 25$) **or** any clear indication that as the 13 does not fit in Bin 1 then $x + 13 > 40$ (condone for this mark just stating that $x > 27$)

dB1: correct answer of $x = 28$ – this mark is dependent on the previous B mark **and the correct upper bound of 28 in (b)** (so if $x = 28$ stated with no justification then no marks). As a minimum accept $x + 13 > 40$ therefore $x > 27$ so $x = 28$ (just '$x > 27$ so $x = 28$' scores **B1 B0** unless explicit mention of where $x > 27$ has come from)

SC in (e): if **B0B0** then award **B1B0** for the first 7 values placed correctly in bins just once (ignore the other values)

Bin 1: x
Bin 2: 23 15
Bin 3: 22 18
Bin 4: 20 13

Question	Scheme	Marks	AOs
5.(a)	**(i)** Route must start at F and finish at J therefore need to consider pairings of the nodes F, C, D and G	M1	3.1b
	CD + FG = 82 + 28 = 110 CF + DG = 71 + 23 = 94 CG + DF = 59 + 22 = 81	A1 A1	1.1b 1.1b
	Repeated roads: CB, BA, AG, and DF	A1	2.2a
	(ii) Length of route = 423 + 81 = 504 (miles)	A1ft	2.2a
		(5)	
(b)	Prim's starting at A: AG, AF, DF, AB, BC, CJ, HJ, EJ	M1 A1 A1	1.1b 1.1b 1.1b
		(3)	
(c)	NNA starting at G: G – A – F – D – B – C – J – H – E – G 8 + 20 + 22 + 65 + 17 + 10 + 12 + 30 + 87 = 271	M1 A1	1.1b 1.1b
		(2)	
(d)	(141 – 8) + 8 + 23 = 164 (miles)	M1 A1	3.1b 2.2a
		(2)	
(e)	GAFD **GA** BCJH **J** E **JCBA** G	B1	3.2a
		(1)	
			(13 marks)

Notes for Question 5

(a)

M1: The correct three pairings of the correct four nodes (F, C, D and G)

A1: Two rows correct including pairings **and** totals

A1: All three rows correct including pairings **and** totals

A1: CAO (CB, BA, AG, DF only but in any order) – must be stated as edges so **A0** for any of CG or CBAG or CG via B and A

A1ft: 423 + weight of their shortest pairing

(b)

M1: First three arcs correctly chosen in order {AG, AF, DF, …} or first four nodes correctly chosen in order{A, G, F, D,…}. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of one of the tables{1, -, -, 4, -, 3, 2, -, -} so please check the top of the tables carefully

A1: First five arcs correctly chosen in order {AG, AF, DF, AB, BC, …} or all nine nodes correctly chosen in order{A, G, F, D, B, C, J, H, E}. Order of nodes may be seen at the top of one of the tables so for the first two marks accept {1, 5, 6, 4, 9, 3, 2, 8, 7} (do not condone any missing numbers e.g. the number 9 must be above E)

A1: CSO – all arcs correct stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of one of the tables unless the correct list of arcs (in the correct order) is also seen)

(c)

M1: Nearest neighbour route starting at G – must have at least G – A – F – D – B – C – … allow if stated in terms of arcs

A1: CAO on length (271) **and** route (must return to G and can be stated in terms of arcs)

(d)

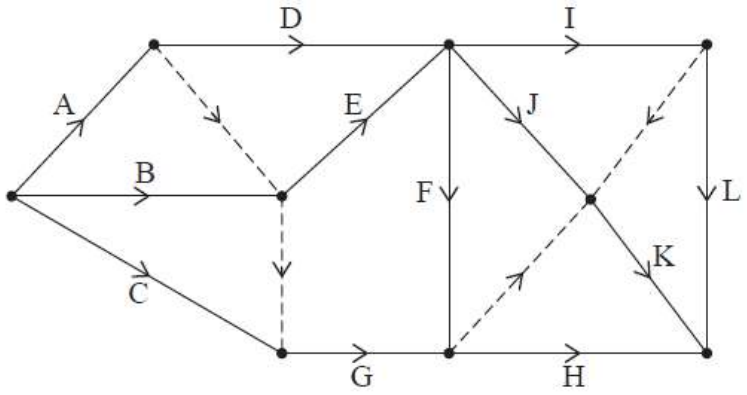
M1: For one of the following

- RMST arcs: AF, DF, AB, BC, CJ, HJ, EJ (stated in any order) – condone AG added too but must be removed at some later stage (e.g. subtracting 8 at some point)
- RMST weight calculation: 20 + 22 + 34 + 17 + 10 + 12 + 18 (not just 20, 22, etc.)
- RMST weight stated: 133 or 141 – 8
- Lower bound stated: 164

A1: CAO (164)

(e)

B1: CAO (GAFD **GA** BCJH **J** E **JCBA** G or in terms of arcs)

Question	Scheme	Marks	AOs
6. (a)		M1 A1 A1 A1 A1	2.1 1.1b 1.1b 1.1b 1.1b
		(5)	
(b)	Critical activities: B, E, I and L	B1	2.2a
		(1)	
(c)	(Earliest start for F is 12 and latest finish is 18 therefore if the total float is 2) the duration of F is 4 (hours)	B1	2.2a
		(1)	
(d)	(i) Maximum possible total float for activity K is $5 - y$	B1	3.1b
	(ii) Total float for activity J is $12 - x - y$	B1	2.2a
		(2)	
			(9 marks)

Notes for Question 6

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. D and E labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F **so use the table below for checking as there a number of acceptable answers**. **Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity C is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore lack of arrows on the activities for the first four marks only (but assume that they are in the 'correct' direction for checking purposes)

(a)

M1: At least nine activities (labelled on arc), one start, at least two dummies placed

A1: Activities A, B, C, D, E and the dummy (+ arrow) at the end of A dealt with correctly

A1: Activities G, F, I and J and the dummy (+ arrow) at the beginning of G dealt with correctly

A1: Activities H and L dealt with correctly

A1: CSO – Final two dummies + arrows and activity K dealt with correctly, all arrows present for every activity with one finish and no additional dummies

Please check all arcs carefully for arrows – if there are no arrows on dummies then M1max
Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

Activity	A	B	C	D	E	F	G	H	I	J	K	L
IPA	-	-	-	A	A, B	D, E	A, B, C	F, G	D, E	D, E	F, G, I, J	I

(b)

B1: CAO (B, E, I and L only)

(c)

B1: CAO (4)

(d)(i)

B1: CAO ($5 - y$)

(d)(ii)

B1: CAO ($12 - x - y$)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>7.(a)</td><td>Maximise $P = 8x + 20y + 40z$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>$4x + 8y + 15z \leqslant 9000$
$x + 5y + 12z \geqslant 3600$</td><td>M1
A1</td><td>3.3
1.1b</td></tr>
<tr><td></td><td>$x + y + z \geqslant 1600$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td>$-x + 3y \leqslant 0$
$(x, y, z \geqslant 0)$</td><td>B1</td><td>3.3</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
<tr><td>(b)</td><td>$4x + 8y + 15z + s_1 = 9000$
$x + 5y + 12z - s_2 + a_1 = 3600$
$x + y + z - s_3 + a_2 = 1600$
$-x + 3y + s_4 = 0$</td><td>B1

B1</td><td>1.1b

2.5</td></tr>
<tr><td></td><td>$I = -(a_1 + a_2)$ where $a_1 = 3600 - x - 5y - 12z + s_2$ and
$a_2 = 1600 - x - y - z + s_3$</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$I - 2x - 6y - 13z + s_2 + s_3 = -5200$ and $P - 8x - 20y - 40z = 0$</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td>e.g.
<table>
<tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>a_1</th><th>a_2</th><th>Value</th></tr>
<tr><td>s_1</td><td>4</td><td>8</td><td>15</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>9000</td></tr>
<tr><td>a_1</td><td>1</td><td>5</td><td>12</td><td>0</td><td>−1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>3600</td></tr>
<tr><td>a_2</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>−1</td><td>0</td><td>0</td><td>1</td><td>1600</td></tr>
<tr><td>s_4</td><td>−1</td><td>3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>P</td><td>−8</td><td>−20</td><td>−40</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>I</td><td>−2</td><td>−6</td><td>−13</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>−5200</td></tr>
</table></td><td>M1
A1</td><td>3.4
2.2a</td></tr>
<tr><td></td><td></td><td>(6)</td><td></td></tr>
</table>

(c)	<table><tr><th>b.v.</th><th>x</th><th>y</th><th>z</th><th>s_1</th><th>s_2</th><th>s_3</th><th>s_4</th><th>Value</th><th>Row Ops</th></tr><tr><td>s_3</td><td>0</td><td>0</td><td>0</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3}$</td><td>1</td><td>0</td><td>200</td><td>$\frac{1}{3}\mathrm{r}_1$</td></tr><tr><td>z</td><td>0</td><td>$\frac{4}{11}$</td><td>1</td><td>$-\frac{1}{33}$</td><td>$-\frac{4}{33}$</td><td>0</td><td>0</td><td>$\frac{1800}{11}$</td><td>$\mathrm{r}_2-\frac{1}{11}\mathrm{R}_1$</td></tr><tr><td>$x$</td><td>1</td><td>$\frac{7}{11}$</td><td>0</td><td>$\frac{4}{11}$</td><td>$\frac{5}{11}$</td><td>0</td><td>0</td><td>$\frac{18000}{11}$</td><td>$\mathrm{r}_3+\frac{12}{11}\mathrm{R}_1$</td></tr><tr><td>$s_4$</td><td>0</td><td>$\frac{40}{11}$</td><td>0</td><td>$\frac{4}{11}$</td><td>$\frac{5}{11}$</td><td>0</td><td>1</td><td>$\frac{18000}{11}$</td><td>$\mathrm{r}_4+\frac{12}{11}\mathrm{R}_1$</td></tr><tr><td>$P$</td><td>0</td><td>$-\frac{4}{11}$</td><td>0</td><td>$\frac{56}{33}$</td><td>$-\frac{40}{33}$</td><td>0</td><td>0</td><td>$\frac{216000}{11}$</td><td>$\mathrm{r}_5+\frac{56}{11}\mathrm{R}_1$</td></tr></table>	B1 M1 A1 A1 B1	1.1b 2.1 1.1b 1.1b 2.4
		(5)	
(d)	(i) £20 500	B1	3.4
	(ii) 1125 + 5(375) + 12(100) = (4200 minutes so) 70 (hours)	B1	2.2a
		(2)	
(e)	e.g. there is no guarantee that all the books will be sold	B1	3.5b
		(1)	
			(19 marks)

Notes for Question 7

(a)

B1: Correct objective function $(8x + 20y + 40z)$ plus 'maximise' or 'max' but not 'maximum' (and '$P =$' is not required)

M1: Either one correct inequality (need not be simplified) **or** both $4x+8y+15z \leqslant k_1$ and $x+5y+12z \geqslant k_2$ where $k_1, k_2 > 0$

A1: Both correct ($4x+8y+15z \leqslant 9000$, $x+5y+12z \geqslant 3600$) – allow equivalent answers (provided 4 terms only and integer coefficients e.g. $2x+10y+24z-7200 \geqslant 0$)

B1: CAO ($x+y+z \geqslant 1600$ oe provided 4 terms only and integer coefficients)

B1: CAO ($3y \leqslant x$ oe provided 2 terms only and integer coefficients)

(b) Note in this part that the numbering of the suffices for the slack, surplus and artificial variables seen in both the candidate's equations and Simplex tableau will most likely be different from what is seen in the MS (e.g. $4x+8y+15z+s_3=9000$ is correct). The correct values in the rows of the tableau do NOT imply the first four marks (the question explicitly asked for the constraints as equations and the rows for the two objectives to be explicitly stated)

B1: Any two correct inequalities converted into equations correctly (condone the same letter, say s_3, being used twice) – any equivalent forms of the correct equations are acceptable (e.g. variables do not need to be on the same side)

B1: All four correct equations (using distinct slack, surplus and artificial variables) – any equivalent forms of the correct equations are acceptable

M1: Using $I = -(a_1 + a_2)$ with their expressions for a_1 and a_2 (allow slips in forming *I* from their two expressions for the two artificial variables) but must be a clear intention to calculate $I = -(a_1 + a_2)$) – must be **exactly two** artificial variables in their expression for *I* for this mark

A1: CAO for *I* and *P* (must be stated as $P - 8x - 20y - 40z = 0$ (**A0** if = 0 missing) and $I - 2x - 6y - 13z + s_2 + s_3 = -5200$ - so variables on one side and constant on the other)

M1: setting up initial tableau – **all** six rows complete (with no blanks) and two correct rows (but ignore b.v. column for this mark)

A1: CAO (any equivalent correct form) - **note that the candidate's order in which the rows appear in the tableau (and choice of letter to represent the slack, surplus and artificial variables) may be different – check to ensure that the basic variable column is consistent with their choice of lettering for the slack and artificial variables**

(c)

B1: Pivot (top) row completely correct including change of b.v. (but not 'Row Ops' column)

M1: All **values** in one of the non-pivot rows correct (so ignore b.v. column and 'Row Ops' column) **or** one of the 'non zero and one' columns (which are y, s_1, s_2 or Value) correct (must have pivoted on the correct value)

A1: Row operations used correctly at least twice, i.e. two of the 'non zero and one' columns (s_1, s_2, y or Value) correct

A1: CAO **all** values including b.v. column – ignore 'Row Ops' column for this mark

B1: Correct row operations stated – alternatives are $\frac{1}{3}\mathrm{r}_1, \mathrm{r}_2 - \frac{1}{33}\mathrm{r}_1, \mathrm{r}_3 + \frac{12}{33}\mathrm{r}_1, \mathrm{r}_4 + \frac{12}{33}\mathrm{r}_1, \mathrm{r}_5 + \frac{56}{33}\mathrm{r}_1$

(d)(i)

B1: CAO (20 500)

(ii)

B1: CAO (70 only)

(e)

B1: CAO – must **explicitly** mention the fact that it is possible that not all the books will be sold. Ignore reasons for why, provided they do not relate to the publisher producing less than the optimal number of books

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